# *Expected Return & Volatility-Manged Portfolio*

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***Abstract:*** How to increase the investment return while holding the constant risk is long-time research for investors and academic finance economists to study. In this paper, we propose an expected return & revised portfolio by managing volatility, which can produce significant positive alphas and increase the Sharpe ratio, showing that our expected return & revised portfolio by managing volatility performs better than the volatility-managed portfolio produced by Alan Moreira and Tyler Muir[1].

## 1.Introduction

During a quite long time, some economists believed that aggregate stock returns could be predictable by studying valuation ratios, such as Graham and Dodd[2], who asserted that if a stock had higher valuation ratios but lower stock price compared to other stocks that had the relatively same perspective, this stock was an undervalued stock and should outcome a relatively higher subsequent return. But this idea was not the mainstream among academic finance economics in a long time until some authors found that valuation ratios have a crucial feature that is positively correlated with stock returns. That means subsequent returns are predicable by valuation ratios at longer horizons. Rozeff[3] indicated that dividend yields were associated with and could be used to predict future stock returns, which meant the higher dividend yield was, the higher future stock returns would be. Compared with the method of predicting future stock returns by using historical realized returns, the method of dividend yields has lower bias, mean square error and mean absolute error. At the same time, Fama and French[4] showed that long term stock returns could be more precisely predicted by dividend yield, compared with predicting short term stock returns. In 1988, a dividend-ratio model introduced by Campbell and Shiller[5] showed the relationship between the dividend and stock price. Meanwhile, another paper written by Campbell and Shiller[6] in the same year asserted a theory that long-term horizon stock returns were easily predictable by studying historical information about real earnings, which could help investors to accurately forecast the present values of future dividends.

Based on the theory that subsequent returns are predicable by valuation ratios at longer horizons, Fama and French introduced a three-factor asset pricing model[7], which included the return on the value-weight (VW) market portfolio (*Mkt*), the return on small size stocks minus return on big size stocks (*SMB*), and return on high B/M stocks minus return on low B/M stocks (*HML*), and then added two more factors, the return on robust profitability stocks minus return on weak profitability stocks (*RMW*) and the return on low investment stocks minus return on high investment stocks (*CMA*), to a five-factor asset pricing model[8], which had more explained power than three-factor asset pricing model. During this period, Hou, Xue and Zhang[9] also produced their intuitive four-factor model, which included the return on the value-weight (VW) market portfolio (*Mkt*), the return on small size stocks minus return on big size stocks (*ME*), the return on low investment stocks minus return on high investment stocks (*I/A*), and the return on robust profitability stocks minus return on weak profitability stocks (return on equity, *ROE*). This asset pricing model provides an asset pricing theory from a different perspective.

From investment, no matter in area of bond, stock, currency, and so on, measuring, predicting and controlling risk is quite an important part for every investor. A rational investor seeks maximum return in constant risk. Therefore, during the past decades, there were a lot of academic finance economists focusing on studying the area of measuring and predicting investment risk. Bollerslev, Hood, Huss and Pedersen[10] introduced a new risk evaluating model based on a utility-based framework for investors to measuring risk. By using this risk evaluating model, investors can get remarkable economic profits. In recent years, some economists have come out with a model that is risk parity, which is an asset allocation concept that allocates the same risk weight to different assets in a portfolio. Rocalli and Weisang[11] analyzed a model that can break up the risk of the portfolio into different risk factors to let the investment portfolio achieve diversified risk. In the area of econometrics, Mercuri and Rroji[12] used the tool of the Independent Component Analysis to linearly decompose portfolio risk into portfolio risk factors. Asnes, Frazzini and Pedersen[13] showed the relationship between the leverage aversion and risk parity. Therefore, managing volatility is an essential skill for investors. Alan Moreira and Tyler Muir produce an effective volatility-managed portfolio that can lead investors to invest properly. This portfolio shows that rational investors should follow the rule, that is, taking more risk when the volatility is low, and taking less risk when the volatility is high. In this pattern, standard mean-variance investors can easily manage their risk to maximize their profits. But we find there are still some factors that cannot be explained by this portfolio. So, I try to improve this portfolio.

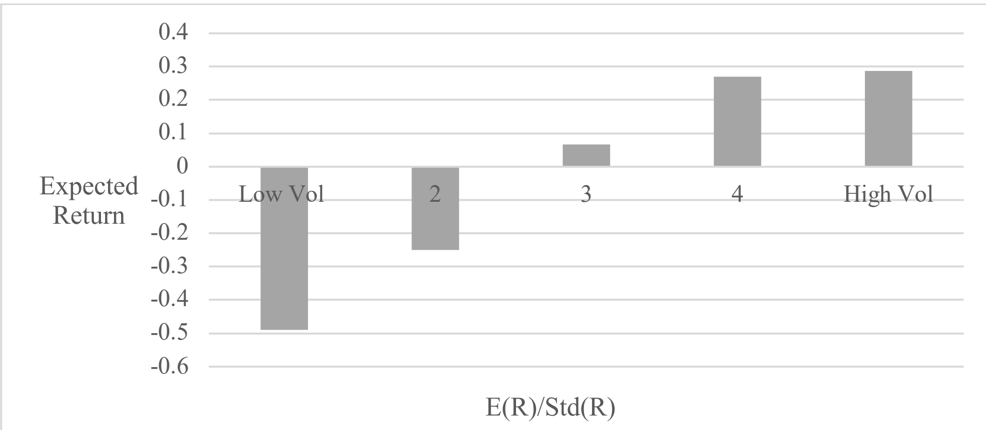


Figure 1: Expected Return classified by managed portfolio.

As we know, a professional investor always changes her or his portfolio allocation based on the theory of the mean-variance trade-off, which shows that at a constant volatility point, the investor should always put the portfolio in the point that can guarantee the maximum profit. As the variance is predictable in the near future, our expected return & revised portfolio by managing volatility should let the mean-variance investors get large alphas and high Sharpe ratios from their investment.

Figure 1 provides intuition for our expected return & revised portfolio by managing volatility. From the expected return & revised portfolio by managing volatility strategy, I grouped data of days by the previous day' s realized volatility and expected return and plotted a graph to show the relation between expected return and expected return divided by realized variance. As we can see, there is a strong relationship between expected return and expected return divided by realized variance. This graph implies that low realized volatility is attractive to standard mean-variance investors, while high realized volatility is unattractive to standard mean-variance investors. Therefore, timing volatility is important for standard mean-variance investors. Hence, standard mean-variance investors should take less risk when volatility is high, and vice versa.

We computed the daily time series of realized volatility and expected return to sort the following daily expected return divided by realized variance into several buckets. The highest, "high vol", represents the highest 20% of expected return divided by realized variance days. The vertical coordinate represents the expected returns. This graph shows a strong relationship between expected return and expected return divided by realized variance.

In summary, we have constructed an expected return & revised portfolio by managing volatility, which replaces the factor $\hat{\sigma}_t^2(f)$ to $\hat{\sigma}_t(f)$ in volatility-managed portfolio produced by Alan Moreira and Tyler Muir. We tested the robustness of the expected return & revised portfolio by managing volatility by running time-series regression in the perspective of single factors, which include Mkt, CMA, HML, SMB, RMW, I/A, ME, and ROE, and of multifactor portfolios, which include the Fama and French's 2015 model[8] and the four-factor model of Hou, Xue and Zhang. Based on the results of the test, we show that the expected return & revised portfolio by managing volatility has great statistical significance and can explain more factors than the original volatility-managed portfolio. That means the expected return & revised portfolio by managing volatility can help investors to manage risk by using realized variance to improve the performance of the investment.

This paper organized as follows. We document our main empirical results in Section I, where Section A is data description, Section B is portfolio formation, Section C is empirical methodology, Section D is Single-factor Portfolio, and Section E is multifactor portfolio. Section II is the conclusion.

## 2. Data Description

First of all, I used daily data on *Mkt, SMB, HML, RMW* and *CMA* from Kenneth French's website. These five factors are included in their five-factor asset pricing model. Then, we included daily data on *Mkt, ME, I/A* and *ROE* from Hou, Xue, and Zhang's website. These four factors are all constructing their four-factor model.

## 3. Method

We constructed the portfolio, as Alan Moreira and Tyler Muir did, by scaling the excess return compared to its conditional variance, but replacing the constant factor c with $\hat{r}_t$, and for making the portfolio even more simple, replacing the factor $\hat{\sigma}_t^2(f)$ with $\hat{\sigma}_t(f)$. Then, the portfolio is

$$f_{t+1}^{r_t,\sigma} = \frac{\hat{r}_t}{\hat{\sigma}_t(f)} f_{t+1} \quad (1)$$

Where $f_{t+1}$ is the excess return for the buy-and-hold portfolio, $\hat{\sigma}_t(f)$ is the portfolio's realized standard deviation, and $\hat{r}_t$ is the expected value of each factor.

We constructed the expected value of each factor by simply using the previous daily value,

$$\hat{r}_t = \frac{\sum_{d=1/22}^{1}(r_{t+d})}{22}. \quad (2)$$

For making the portfolio more simple, we used the previous daily realized variance as the factor to calculate the conditional variance,

$$\hat{\sigma}_t(f) = \sum_{d=1/22}^{1}(f_{t+d} - \frac{\sum_{d=1/22}^{1} f_{t+d}}{22}) \quad (3)$$

We ran a time-series regression based on the different factors,

$$f_{t+1}^{r_t,\sigma} = \alpha + \beta f_{t+1} + \varepsilon_{t+1} \quad (4)$$

A positive or negative but statistically insignificant α means the result can be explained by the original factors, that is the portfolio does not increase the Sharpe ratio. Whereas, a positive α implies that the portfolio increases the Sharpe ratio by the factors other than the factors included in the original volatility-managed portfolio.

## 4. Empirical Results

Firstly, we ran the regression factor by factor to test the robustness on factors.

Table 1 shows the result of the original volatility-managed portfolio, which is using the constant proxy c. We can see statistically insignificant intercepts (α's) in most cases, even if positive. The market factor has an only alpha of 0.009% and a beta of 0.942, but it is statistically insignificant under 5% confidence interval. While most alphas are positive but small and statistically insignificant, the largest alpha is the factor for return on equity, *ROE*, which is 0.039% and is one of the three statistically significant factors, the other two of which are *I/A* and *CMA*. Above all, the original volatility-managed portfolio cannot increase the Sharpe ratio.

We use dependent variable $f^{\sigma}$ and independent variable $f_{t+1}^{\sigma}$. The data are daily from 1963 to 2020 for every factor from the 2015 model, and 1967 to 2019 for every factor from the four-factor model. Robust standard errors in parentheses. * p<0.1, ** p<0.05, *** p<0.01.

Table 1: The Original Volatility-Managed Portfolio.

| | $Mkt^{\sigma}$ | $SMB^{\sigma}$ | $HML^{\sigma}$ | $RMW^{\sigma}$ | $CMA^{\sigma}$ | $ME^{\sigma}$ | $I/A^{\sigma}$ | $ROE^{\sigma}$ |
|---|---|---|---|---|---|---|---|---|
| *Mkt* | 0.942*** | | | | | | | |
| | (0.020) | | | | | | | |
| *SMB* | | 1.915*** | | | | | | |
| | | (0.047) | | | | | | |
| *HML* | | | 1.837*** | | | | | |
| | | | (0.033) | | | | | |
| *RMW* | | | | 2.505*** | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | (0.047) | | | | |
| *CMA* | | | | | 2.649*** | | | |
| | | | | | (0.050) | | | |
| *ME* | | | | | | 1.861*** | | |
| | | | | | | (0.044) | | |
| *I/A* | | | | | | | 2.593*** | |
| | | | | | | | (0.061) | |
| *ROE* | | | | | | | | 2.351*** |
| | | | | | | | | (0.034) |
| Constant | 0.009* | 0.004 | 0.012** | 0.017*** | 0.002 | 0.001 | 0.013*** | 0.039*** |
| | (0.005) | (0.004) | (0.005) | (0.005) | (0.005) | (0.004) | (0.005) | (0.005) |
| Observation | 14,263 | 14,263 | 14,263 | 14,263 | 14,263 | 13,318 | 13,318 | 13,318 |
| Adj. $R^2$ | 0.731 | 0.805 | 0.704 | 0.709 | 0.766 | 0.815 | 0.778 | 0.762 |

Then, we ran regression repeatedly but change the constant c to $\hat{r}_t$. We find that there are statistically significant and positive intercepts (α's) even under 1% confidence interval in almost every case, except the investment factor, *I/A*, having a negative alpha, which is -0.009%. The market factor has an alpha of 0.007% and a beta of -0.07, and it is statistically significant even under 1% confidence interval. The largest alpha is the factor for the size factor, *SMB*, which is 0.016%, and this factor has a beta of -0.014. The results show that the original factors cannot fully explain the increase of the Sharpe ratio as our portfolio does. That means the expected return & revised portfolio by managing volatility can statistically increase the Sharpe ratio. The results show in Table 2.

Table 2: Expected Return & Revised Portfolio by Managing Volatility Alphas.

| | $Mkt^{r_t,\sigma}$ | $SMB^{r_t,\sigma}$ | $HML^{r_t,\sigma}$ | $RMW^{r_t,\sigma}$ | $CMA^{r_t,\sigma}$ | $ME^{r_t,\sigma}$ | $I/A^{r_t,\sigma}$ | $ROE^{r_t,\sigma}$ |
|---|---|---|---|---|---|---|---|---|
| *Mkt* | -0.070*** | | | | | | | |
| | (0.008) | | | | | | | |
| *SMB* | | -0.014 | | | | | | |
| | | (0.011) | | | | | | |
| *HML* | | | 0.040*** | | | | | |
| | | | (0.010) | | | | | |
| *RMW* | | | | 0.059*** | | | | |
| | | | | (0.010) | | | | |
| *CMA* | | | | | 0.099*** | | | |
| | | | | | (0.008) | | | |
| *ME* | | | | | | -0.005 | | |
| | | | | | | (0.010) | | |
| *I/A* | | | | | | | 0.006 | |
| | | | | | | | (0.007) | |
| *ROE* | | | | | | | | 0.074*** |
| | | | | | | | | (0.008) |
| Constant | 0.007*** | 0.016*** | 0.014*** | 0.007*** | 0.006*** | 0.013*** | -0.009*** | 0.010*** |
| | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| Observat-ion | 14,263 | 14,263 | 14,263 | 14,263 | 14,263 | 13,318 | 13,318 | 13,318 |

| Adj. $R^2$ | 0.078 | 0.002 | 0.015 | 0.035 | 0.103 | 0.000 | 0.000 | 0.053 |
|---|---|---|---|---|---|---|---|---|

We use dependent variable $f^{\sigma}$ and independent variable $f_{t+1}^{r_t,\sigma}$. The data are daily from 1963 to 2020 for every factor from the 2015 model, and 1967 to 2019 for every factor from the four-factor model. Robust standard errors in parentheses. * p<0.1, ** p<0.05, *** p<0.01.

## 5. Robust test

Furthermore, we consider how the expected return & revised portfolio by managing volatility works on the multifactor environment. We built the portfolios by combining multiple factors based on different models. We choose weights so that the weights of factors in our multifactor portfolio could be as same as the theories of Fama-French's 2015 model and of the four-factor model produced by Hou, Xue, and Zhang. That means our multifactor expected return & revised portfolio by managing volatility could be tested both for the single individual factors and for the wide range of assets pricing models priced by Fama-French's 2015 model and assets pricing model produced by Hou, Xue, and Zhang. The alpha of our multifactor portfolio is a good sign to measure how good our portfolio works in the frame of the theories. Specifically, a positive and statistically significant alpha implies that our expected return & revised portfolio by managing volatility increases Sharpe ratios compared to the best Sharpe ratio can be achieved by professional investors who consider the multiple factors.

We constructed the multifactor expected return & revised portfolio by managing volatility showed as follows. We let $F_{t+1}$ represent the vector of factor returns and $a$ represent the static weight that can let the portfolio produce the maximum alpha. We defined the portfolio as $f_{t+1}^{mul} = a' F_{t+1}$. Then we constructed

$$f_{t+1}^{mul,r_t,\sigma} = \frac{\hat{r}_t}{\hat{\sigma}_t\,(f_{t+1}^{mul,r_t})} f_{t+1}^{mul,r_t} \quad (5)$$

Where again $\hat{r}_t$ is the expected value of each factor and $\hat{\sigma}_t\,(f_{t+1}^{mul,r_t})$ is the realized standard deviation.

In Table 3, we show that our expected return & revised portfolio by managing volatility has positive and statistically significant alpha for all combinations of factors I consider including not only the Fama-French's 2015 model but also the assets pricing model produced by Hou, Xue, and Zhang. Specifically, our portfolio can increase the Sharpe ratio even in a multifactor environment.

In Panel A, we form the multifactor portfolios by using various combinations of factors that are included in Fama-French's 2015 model. In Panel B, we form the multifactor portfolios by using various combinations of factors that are included in the four-factor model produced by Hou, Xue, and Zhang. We then run the time-series regression on each of these two models and report alphas from regressing the expected return & revised portfolio by managing volatility on the Fama-French's 2015 model[8] and assets pricing model produced by Hou, Xue, and Zhang portfolio. Robust standard errors in parentheses. * p<0.1, ** p<0.05, *** p<0.01.

Table 3: Multifactor Portfolios.

| Panel A: Multifactor Portfolios (Fama-French five factors) | | | | | |
|---|---|---|---|---|---|
| | $Mkt^{r_t,\sigma}$ | $SMB^{r_t,\sigma}$ | $HML^{r_t,\sigma}$ | $RMW^{r_t,\sigma}$ | $CMA^{r_t,\sigma}$ |

| | | | | | |
|---|---|---|---|---|---|
| *Mkt* | -0.070*** | -0.008*** | -0.020*** | -0.006*** | -0.007*** |
| | (0.009) | (0.003) | (0.004) | (0.002) | (0.001) |
| *SMB* | -0.010 | -0.016 | -0.016*** | 0.001 | -0.001 |
| | (0.009) | (0.011) | (0.006) | (0.003) | (0.004) |
| *HML* | 0.001 | -0.003 | 0.013 | -0.008 | 0.016*** |
| | (0.011) | (0.007) | (0.013) | (0.005) | (0.003) |
| *RMW* | -0.020* | -0.000 | 0.007 | 0.053*** | 0.013** |
| | (0.010) | (0.008) | (0.008) | (0.009) | (0.005) |
| *CMA* | 0.020 | 0.017** | 0.055*** | 0.051*** | 0.079*** |
| | (0.014) | (0.008) | (0.012) | (0.006) | (0.008) |
| Constant | 0.007*** | 0.016*** | 0.014*** | 0.007*** | 0.006*** |
| | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) |
| Observations | 14,263 | 14,263 | 14,263 | 14,263 | 14,263 |
| Adj. $R^2$ | 0.080 | 0.007 | 0.052 | 0.063 | 0.111 |

Panel B: Multifactor Portfolios (Hou, Xue, and Zhang)

| | $ME^{r_t,\sigma}$ | $I/A^{r_t,\sigma}$ | $ROE^{r_t,\sigma}$ | $ROE^{r_t,\sigma}$ |
|---|---|---|---|---|
| *Mkt* | -0.058*** | -0.010*** | -0.015*** | 0.004** |
| | (0.009) | (0.002) | (0.005) | (0.002) |
| *ME* | -0.002 | -0.008 | 0.095*** | 0.001 |
| | (0.010) | (0.010) | (0.020) | (0.003) |
| *I/A* | 0.035*** | 0.017** | -0.011* | 0.025*** |
| | (0.010) | (0.007) | (0.007) | (0.005) |
| *ROE* | -0.019** | -0.013** | 0.005 | 0.074*** |
| | (0.009) | (0.005) | (0.004) | (0.008) |
| Constant | 0.006*** | 0.014*** | -0.010*** | 0.009*** |
| | (0.002) | (0.001) | (0.001) | (0.001) |
| Observations | 13,318 | 13,318 | 13,318 | 13,318 |
| Adj. $R^2$ | 0.062 | 0.007 | 0.108 | 0.057 |

## 6. Conclusion

This paper illustrates that expected return & revised portfolio by managing volatility produces a good way for investors to get a large alpha and increase the Sharpe ratio. We run the time-series regression by using the daily data which has a sample period from 1963 to 2020 for *Mkt*, *SMB*, *HML*, *RMW*, and *CMA* from Fama-French's 2015 model, and that of the sample period from 1967 to 2019 for *ME*, *I/A*, and *ROE* from Hou, Xue and Zhang assets pricing model to test the robustness of this portfolio. The results show that this portfolio has great statistical significance not only in every single factor but also in the Fama-French's 2015 model and Hou, Xue and Zhang assets pricing model. Moreover, our expected return & revised portfolio by managing volatility is robust in other factors, providing an easy way for investors to follow to improve the performance of the investment.

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